A NOTE ON BALANCED n-ARY DESIGNS

By

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1. Introduction

Balanced n-ary designs having constant block sizes and equal replications in general were first introduced by Tocher [11]. However as pointed out by Pearce [8], while experimenting with new strains of crop severely limited by the available amount of seed together with one or two standard varieties with no limitation on seeds or while conducting nutritional experiments with unequal numbers of animals in available litters or in situations similar to these equal block size and equal replication may act as serious obstacles. The problem then is to obtain balanced designs either with unequal block sizes or with unequal replications or with both these properties. In these respects appreciable results have been obtained by Kulshreshtha *et al.* (5), John [4] and Nigam *et al.* [7]. The number of blocks of most of the available designs is generally very large thus rendering them useless in practical situations.

It is proposed here to construct some balanced n-ary designs with the help of known arrangements such as orthogonal arrays, partially balanced arrays, balanced matrices and balanced incomplete block designs.

2. Some Preliminary Concepts

n-ary design has been defined by Murthy and Das [5] orthogonal arrays, partially balanced arrays and balanced matrices are defined respectively by Bush [1], Chakrabarti [2] and Shah [9].

Definition 2.1. Two BIB designs with incidence matrices N_1 and N_2 both of order $v \times b$ are said to be associable if

(*a*) $N_1 + N_2$ is a zero-one matrix and

(*b*) $N_1 N_2' = \times (JJ' - I)$ where $\times$ is a scalar and J is a column vector of all unities.

From the definitions it can be easily understood that, if one symbol is or two or more symbols are replaced by 1 and the rest by 0 in either an orthogonal array, a partially balanced array of strength 2 or a balanced matrix, what is obtained will be the incidence matrix of a balanced incomplete block design. Further, if the symbols of any of these matrices are classified into exclusive sets and the symbols in any one set are replaced by 1 and those of other by zero and the process is repeated with respect to every set, the matrices so obtained will be incidence matrices of balanced incomplete block designs (BIBD) which are associable.

3. Proper n-ary Designs

A proper balanced n-ary design has blocks of equal sizes.

Theorem. Let N_i, $i=1, 2, \ldots\ldots k$ be k associable BIB designs with parameters v, b, r_i, k_i, λ_i. If Pi, $i=1, 2, \ldots\ldots k$ are non-negative integers, then

$N=p_1N_1+\ldots\ldots+p_kN_k$ will be a proper balanced n-ary design if the largest among the p_i's is $n-1$.

Proof. Since N_i and N_j are associable $N_iN_j'=d_{ij}D$ where d_{ij} is a positive integer and $D=(JJ'-I)$. The row totals of N are equal and so are its column totals.

Also $NN'=rI+\lambda(JJ'-I)$ where it is easy to show that

$$r=\sum p_i^2 \; r_i \text{ and } \lambda=\sum_i p_i^2 \; \lambda_i+\sum_{\substack{ij \\ i\neq j}} p_ip_j \, d_{ij}$$

Thus the C matrix of N has all diagonal elements equal and all off diagonal elements also equal.

Summary

A method of construction of proper balanced n-ary designs from associable designs (BIB) has been suggested.

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